CONFIDENTIAL*

950/1, 954/1 **STPM 2012**

MATHEMATICS S
PAPER 1 (SET 1)
MATHEMATICS T
PAPER 1 (SET 1)

Three hours

PEPERIKSAAN PERCUBAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
NEGERI PAHANG DARUL MAKMUR 2012

Instructions to candidates:

Answer **all** *questions.*

Answers may be written in either English or Bahasa Malaysia.

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures,

or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae and graph paper are provided.

This question paper consists of 5 printed pages.

Mathematical Formulae for Paper 1 Mathematics T / Mathematics S :

Logarithms :

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Series :

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Integration :

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

Series:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n \text{, where } n \in \mathbf{N}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots \text{, where } |x| < 1$$

Coordinate Geometry :

The coordinates of the point which divides the line joining (x_1 , y_1) and (x_2 , y_2) in the ratio $m : n$ is

$$\left(\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$$

The distance from (x_1, y_1) to $ax+by+c=0$ is

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Numerical Methods :

Newton-Raphson iteration for $f(x)=0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Trapezium rule :

$$\int_a^b f(x)dx \approx \frac{1}{2}h[y_0 + 2(y_1+y_2+\cdots+y_{n-1}) + y_n] \quad \text{where } y_r = f(a+rh) \text{ and } h = \frac{b-a}{n}$$

Trigonometry :

$\sin(A \pm B) = \sin A\cos A \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

1. If A , B and C are arbitrary sets, show that $[(A'\cap C)\cup(B'\cap C)]\cup(A\cap B\cap C)=C$.[4 marks]

2. Using trapezium rule, with five ordinates, evaluate $\int_1^2 \ln(1+x^2)dx$, giving your answer correct to three decimal places. [4 marks]

3. Solve the following inequalities.
   (a) $\frac{1}{x-5}-\frac{1}{x+2}\geq 0$ [3 marks]
   (b) $|3x-2|<x+5$ [3 marks]

4. Given $e^x y=\sin x$, show that $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$. . [6 marks]

5. The function f is defined by $f(x)=\begin{cases} x^2 & ,\ 0\leq x<1 \\ 3-2x & ,\ 1\leq x<2 \end{cases}$.

   (a) Find $\lim_{x\to1^+} f(x)$ and $\lim_{x\to1^-} f(x)$.
   Hence, show that f is continuous at x = 1. [4 marks]

   (b) Given the function g is periodic with period 2 and $g(x)=f(x)$ for $0\leq x<2$, sketch the graph of g for $-2\leq x<4$. [3 marks]

6. Given the complex number $z=p-3i$, $p\in R$, $i^2=-1$ such that $\frac{z^2}{4-3i}$ is a real number,

   (a) simplify the complex number z^2 in terms of p , [1 marks]

   (b) determine the possible values of p. [4 marks]

   If $p<0$, find

   (c) the real number $\frac{z^2}{4-3i}$, [1 mark]

   (d) the argument of z. [2 marks]

7. The circle C has the equation $3x^2 + 3y^2 - 6x + 12y - 60 = 0$. The equation of the straight line l is $2y = x + 5$.

(a) Find

(i) the coordinates of the centre of C and the radius of C. [3 marks]

(ii) the distance from the centre of C to the straight line l. [2 marks]

(b) Given that the straight line l intersects the circle C at $A(x_1, y_1)$ and $B(x_2, y_2)$ such that $x_1 < x_2$, find

(i) the coordinates of the points A and B,

(ii) the value of k if the points A, B and (0 , k) are collinear. [4 marks]

8. It is given that $\dfrac{3x^4 + 2x^2 + 1}{x^3 + x}$ can be express as $f(x) + \dfrac{1 - x^2}{x^3 + x}$.

(a) Determine $f(x)$. [1 mark]

(b) Express $\dfrac{1 - x^2}{x^3 + x}$ as sum of partial fractions. [4 marks]

(c) Show that $\displaystyle\int_1^2 \frac{3x^4 + 2x^2 + 1}{x^3 + x}\,dx = \frac{9}{2} + \ln\left(\frac{4}{5}\right)$. [4 marks]

9. Expand $\dfrac{\sqrt[4]{1 + ax^2}}{(1 - x)^2}$ in ascending powers of x up to and including the term in x^3. [4 marks]

Given that the first four terms in the above expansion are $1 + 2x + 4x^2 + 6x^3$, find

(a) the value of a , [2 marks]

(b) the set of values of x for which the expansion is valid. [2 marks]

(c) the value of $\sqrt[4]{17}$ correct to three significant figures by taking $x = \dfrac{1}{8}$. [2 marks]

10. The equation of a curve is given by $y = \frac{1}{x} - x^2$.

(a) Show that there exists only one turning point and determine the nature of this point. [4 *marks*]

(b) Determine the respective intervals for which the curve concaves downward and concaves upwards. [4 *marks*]

(c) Sketch the curve showing clearly its asymptote. [3 *marks*]

11. The polynomial $p(x) = 6x^4 + ax^3 + bx^2 + x - 1$, where *a* and *b* are real constants has a factor $(2x - 1)$. The derivative of $p(x)$ with respect to x, $p'(x)$, leaves a remainder -76 when divided by $(x + 1)$.

(a) Find the values of *a* and *b*. [6 marks]

(b) Explain why 1 is a zero of $p(x)$.
Hence, solve the equation $p(x) = 0$. [4 marks]

(c) Find the set of values of x such that $\frac{p(x)}{3x^2 - 2x - 1} \leq 0$. [3 marks]

12. Given that $M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ and $N = \begin{pmatrix} 15 & -1 & -4 \\ -1 & 15 & 4 \\ -4 & 4 & 16 \end{pmatrix}$.

(a) Show that M is a non singular matrix (i.e the inverse of M is defined). [2 marks]

(b) Determine the matrices N – 6M and M(N – 6M).
Hence, find the inverse of *M*. [5marks]

(c) From the inverse of M in (b), find the adjoin of M [2 marks]

(d) Use all the information above, solve the simultaneous equations

$$20x - 10z = -100$$
$$20y + 10z = 300$$
$$-10x + 10y + 20z = 200$$

[4 marks]

===END OF QUESTION PAPER===

CONFIDENTIAL*

954/2 **STPM 2012**

MATHEMATICS T
PAPER 2
SET 1

Three hours

PEPERIKSAAN PERCUBAAN

SIJIL TINGGI PERSEKOLAHAN MALAYSIA
NEGERI PAHANG DARUL MAKMUR 2012

Instructions to candidates:

Answer **all** *questions. Answers may be written in either English or Malay.*

All necessary working should be shown clearly.

Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Mathematical tables, a list of mathematical formulae and graph paper are provided.

Question no.	Full Marks	Marks
1	5	
2	12	
3	7	
4	10	
5	6	
6	10	
7	6	
8	6	
9	10	
10	9	
11	12	
12	7	
	100	

This question paper consists of 6 printed pages.

Mathematical Formulae for Paper 2 Mathematics T :

Logarithms :

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Series :

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Integration :

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left| f(x)\right| + c$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

Series:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n, \textbf{ where } \mathbf{n} \in \mathbf{N}$$

Coordinate Geometry :

The coordinates of the point which divides the line joining (x_1 , y_1) and (x_2 , y_2) in the ratio m : n is

$$\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$$

The distance from (x_1, y_1) to $ax + by + c = 0$ is

$$\frac{\left| ax_1 + by_1 + c\right|}{\sqrt{a^2+b^2}}$$

Maclaurin expansions

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}x^r + \cdots, \textbf{ where } |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1}x^r}{r} + \ldots, -1 < x \le 1$$

Mathematical Formulae for Paper 2 Mathematics T :

Numerical Methods :

Newton-Raphson iteration for $f(x)=0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Trapezium rule :

$$\int_a^b f(x)dx \approx \frac{1}{2}h[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$$

where $y_r = f(a+rh)$ **and** $h = \frac{b-a}{n}$

Correlation and regression :

Pearson correlation coefficient:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

Regression line of *y* on *x* :

$$y = a + b\,x$$

where $$b = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$a = \bar{y} - b\bar{x}$$

Trigonometry

$$\sin(A \pm B) = \sin A \cos A \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

1. Prove that $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 16 \sin\theta \cos^2\theta \cos^2 2\theta$. **[5]**

2. (a) In the diagram 2A, QAPK and RBP are straight lines and PT is a tangent. If ∠KPT =72° and ∠TPR = 65°, find ∠PQR and ∠PRQ . **[4]**

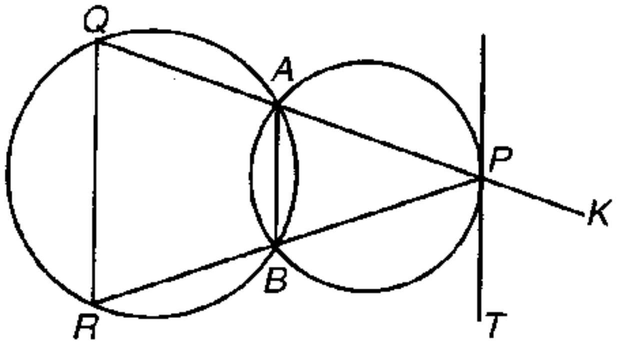
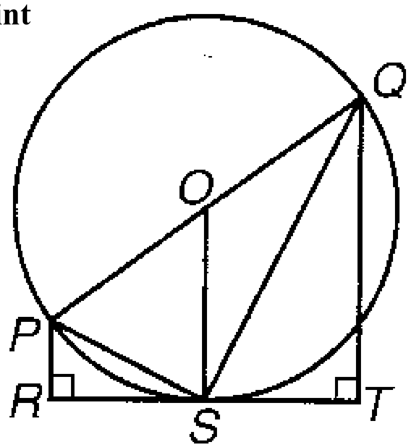


Diagram 2A

Diagram 2B



(b) In the diagram 2B, PQ is a diameter of the circle and S is a point on the circumference of the circle with centre O. Perpendiculars from P and Q meet the tangent through S at points R and T.

(i) Prove that ∠PSR = ∠OSQ. **[3]**

(ii) Prove that ΔPSR and ΔSQT are similar. **[3]**

(iii) Prove that PR · QT = RS · ST . **[2]**

3. (a) If $\lambda \mathbf{i} + 3\mathbf{j}$ and $3\mathbf{i} + (8 + \lambda)\mathbf{j}$ are two parallel vectors find the possible values of λ . **[3]**

(b) Points P , Q and R have positions vectors $3\mathbf{i} - \mathbf{j}$, $2\mathbf{i} + 2\mathbf{j}$ and $-\mathbf{i} - 2\mathbf{j}$. Calculate the scalar product of $\overrightarrow{QP} \bullet \overrightarrow{RP}$, hence find angle QPR . **[4]**

4. By using the substitution $y = vx$, show that the differential equation $y\left(\frac{dy}{dx}\right) = 2y - x$ can be changed into the differential equation $x\left(\frac{dv}{dx}\right) + \frac{(v-1)^2}{v} = 0.$
Hence, solve this differential equation , given that $y = 2$ when $x = 1$, show that $\ln(y - x) = \frac{2x - y}{y - x}$. **[10]**

5. The following set of data shows the age (to the nearest year) of a random sample of 36 people who registered their names to book Perodua Viva in January 2012.

28	**35**	**31**	**32**	**31**	**25**	**34**	**26**	**37**	**33**	**30**	**29**
30	**36**	**48**	**44**	**32**	**37**	**41**	**37**	**39**	**60**	**51**	**41**
37	**37**	**31**	**28**	**43**	**32**	**35**	**41**	**30**	**37**	**50**	**43**

(a) Copy and complete the stem plot below.

25	**3**	
30	**1**	**2**
35	**0**	

[2]

(b) Draw a boxplot for this data and use your box plot to identify the 'outliers' . [4]

6. The height (up to the nearest cm) of 80 Form 6 male students in a school is given in the table below.

Height	**150 – 154**	**155 – 159**	**160 –164**	**165 – 169**	**170 – 174**	**175 – 179**	**180 – 184**
Number of students	**3**	**6**	**11**	**17**	**25**	**12**	**6**

(a) Display this distribution on a histogram. Hence, estimate the median age. [4]

(b) Calculate an estimate for the mean and standard deviation for the data in the above distribution. [5]

(c) Explain why the mean and standard deviation are not necessarily the best statistical representation for this distribution. [1]

7.The probability distribution of a uniform discrete random variable , X is given below .

X	**1**	**2**	**3**	…	…	…	***n***
P (X = *x*)	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	…	…	…	$\frac{1}{n}$

Show that $E(X) = \frac{1}{2}(n+1)$ **and** $Var(X) = \frac{1}{12}(n^2 - 1)$ **. [6]**

8. The lifespan , in hours, of an electrical component is a random variable X with probability density function as follows

$$f(x) = \begin{cases} \frac{1}{100} e^{-\frac{x}{100}} & , x > 0 \\ 0 & , otherwise \end{cases}$$

(a) Calculate the mean lifespan of the electrical components. [5]

(b) Five electrical components are chosen at random and Y is the total lifespan of all the electrical components. Find E(Y). [1]

9. (a) Express 4 cos θ – 3 sin θ in the form R cos (θ ± α) , where R is a positive constant and α an acute angle. Hence, solve the equation 4 cos θ – 3 sin θ = 1 in the interval 0° ≤ θ ≤ 360 °. [4]

(b) State the maximum and minimum values for *y* = 4 cos θ – 3 sin θ and their corresponding angles.
Sketch of the graph *y* = 4 cos θ – 3 sin θ in the interval 0° ≤ θ ≤ 360 °. [4]
Hence, solve 4 cos θ – 3 sin θ ≥ 1 . [2]

10. A gold prospector examines 800 gold bearing ingots each month. The contents of each ingot can be assumed to be independent and the probability of one ingot containing gold is 0. 005.
(a) The prospector is considered having a lucky month if 4 or more ingots examined by him contain gold. By using a suitable approximation , show that the probability a month chosen randomly is a lucky month is 0. 567 (correct to 3 significant figures). [4]

(b) By using a suitable approximation , determine the probability that in 24 months chosen at random, there are more than 12 lucky months. [5]

11. (a) Diameters of a type of steel pipes produced in a factory are normally distributed with mean 0. 95 cm and standard deviation σ cm. If at least 88 % of the steel pipes produced have diameters which are less than 0. 98 cm , find the range of values of σ .[4]

(b) Four runners A , B , C and D ran 100 m each . The time taken, in seconds , by each runner can be considered as independent observations from a normal distribution with mean 14 and standard deviation 0. 2 . A runner, E , ran 400 m. The time taken, in seconds, by E can be considered as an observation from a normal distribution with mean 58 and standard deviation 1. 0 and independent of the times taken by the other runners.

(i) Determine the probability that the time taken by runner E is less than four times the time taken by runner A . [4]

(ii) Determine the probability that the time taken by runner E is less than 3 seconds more than that of the total time by all four runners A , B , C and D. [4]

12. At noon, an observer on a ship A sees another ship B which is 20 km away to the north of the observer's ship A. Ship A is sailing eastwards at a speed of 20 km/h . Ship B is sailing in the directions S 30° E at a speed of 8 km/h.

(a) Find the velocity of B relative to A. [3]

(b) Find the closest distance between A and B. [2]

(c) Find the time when the distance between the two ships is the closest. [2]